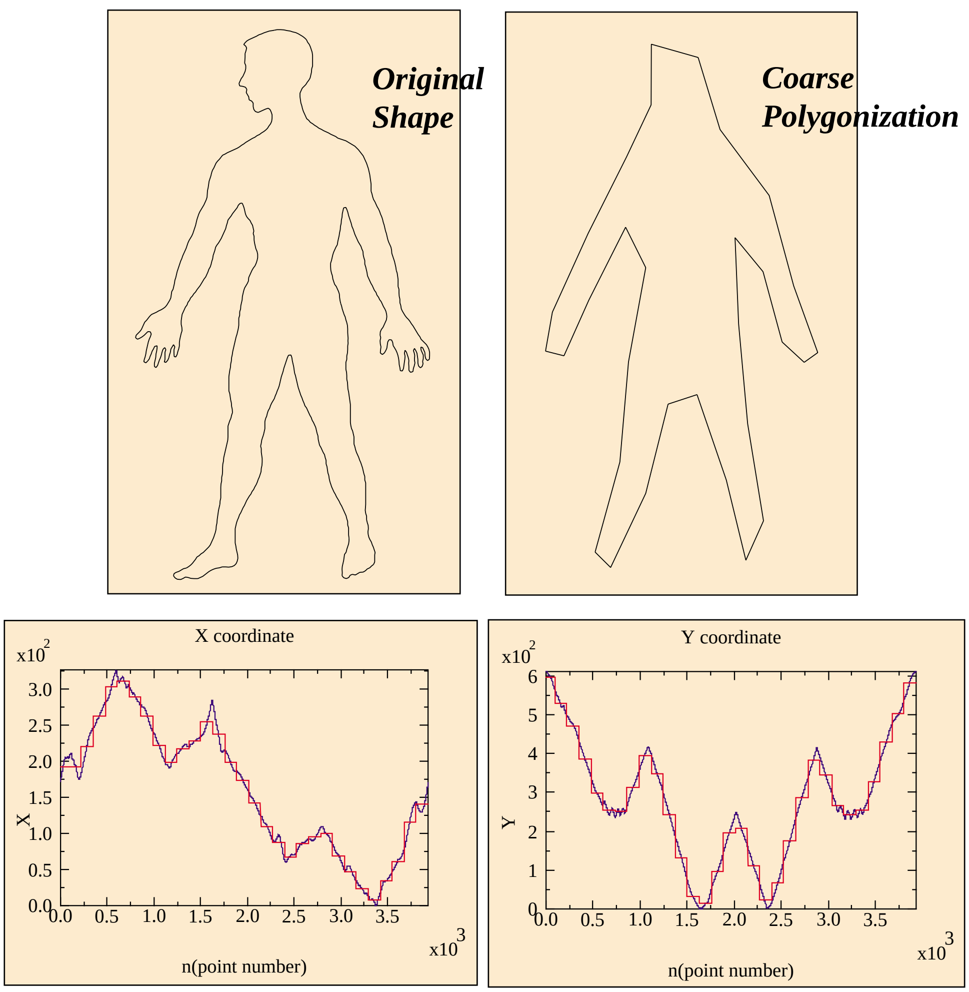
T-DO
Theoretical Division Office

Morphological Analysis of Shapes

Lakshman Prasad and Ramana Rao

The ability to computationally characterize and analyze shapes is crucial to much of Computer Vision, Pattern Recognition, and Image Understanding. The semantics of shape, however, is essentially anthropocentric in most practical applications. Indeed, it is desirable that a robot capable of describing objects in a scene, be able to do so in a way that makes sense to humans. Confronted with a complex shape, we humans are able to rapidly single out the most salient "*geometric events*" (such as kinks and protuberances), and use them to simplify the shape to a *morphological primal sketch*. The shape is then interpreted by analogy, association or even imagination (as in the case of cloud gazers!). Humans also seem to be able to do this in a multiresolutional fashion, picking out the gross forms first and subsequently embellishing them with finer details. Thus, in order to computationally understand shapes, it is necessary to extract *those* geometric cues that are morphologically meaningful. Using Computational Geometry, we have been developing algorithms for morphologically analyzing and characterizing shapes in a multiscale setting.



Coordinates (blue) and Haar approximations (red).

Figure 1.

The notion of *shape* is intimately related to the notion of *contour* or *boundary*.The boundary of a shape has, however, a continuum of points, and as such is not amenable to finite representation or computation. Thus, it is vital to first obtain a discrete representation of its boundary in a morphologically faithful manner; i.e., preserving its structural integrity. If the coordinates of the boundary points of a shape are specified parametrically, then any discretization of the boundary can be thought of as a piecewise constant approximation of the coordinate functions. Also, in order to be morphologically faithful to the shape, the discretization must be scale-adaptive to local variations of the shape's boundary. Now, the discrete Haar wavelet expansion of a function yields piecewise constant approximations of the function at multiple scales. By representing the slowly varying and the rapidly varying parts of the function by Haar approximations at different scales, the function can be approximated to any degree by a piecewise constant function. Using this idea, the coordinate functions of a parametrically specified curve may be jointly and adaptively approximated at varying scales to obtain a discretized representation of the curve (see Figure 1).

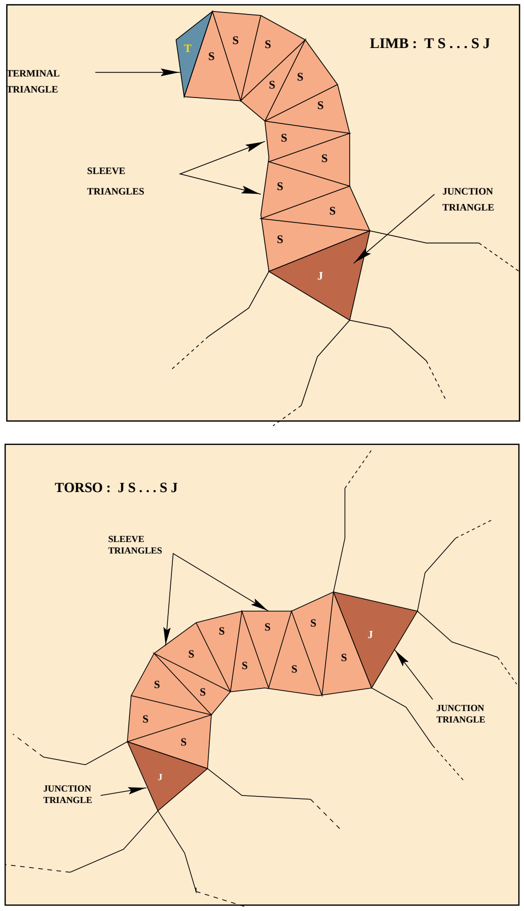


Figure 2.

Next, the fundamental morphological attributes of the shape must be extracted from this representation. We look at a familiar geometric construct—the Delaunay Triangulation (DT)—from a novel perspective: the DT of a discretely sampled shape reveals important morphological information that helps characterize, recognize and analyze the shape. It serves as a natural morphological grid that localizes the structural properties of the shape. Indeed, the DT of a polygonal shape has three kinds of triangles: i) *Terminal triangles* or *T-triangles*, each with two external (i.e., polygonal boundary) edges, marking the termination of a limb or a protrusion of the polygon, ii) *Sleeve triangles* or *S-triangles*, each with one external edge, constituting the sleeve of a limb or protrusion and iii) *Junction triangles* or *J-triangles*, with no external edges, determining a junction or a branching of the polygon. Thus, each kind of triangle carries morphological information about the local structure of the polygon. In any triangulation of a simple polygon, the number Δ_J of J-triangles is related to the number Δ_T of T-triangles by $\Delta_J = \Delta_T + 2g - 2$, where g is the *genus* (i.e., number of holes) of the polygon. We will identify two kinds of chain complexes of triangles in any triangulation of a polygon: i) A *limb* λ is a chain complex of pairwise adjacent triangles, of the form **J S ... S T**, and ii) a *torso* τ is a chain complex of pairwise adjacent triangles, of the form **J S ... S J** (see Figure 2); the J-triangles at the ends of a torso may be the same triangle (as in the case of loops or handles). Chain complexes of the form **T S ... S T** are both limbs *and* torsos. The number of limbs in a triangulation of a polygon is given by $N(\lambda) = \Delta_T + min(1, \Delta_J) - 1$, and the number of torsos is given by

$$N(\tau) = \left\lfloor \frac{3\Delta_J - \Delta_T}{2} \right\rfloor .$$

In the context of the DT of a polygon, however, the above formulae have *real* morphological significance. The limb and torso chain complexes of the DT of a polygon actually *do* correspond to morphological limbs and torsos (i.e., trunks connecting branch points) of the polygon's structure. These features (e.g., holes, limbs, and torsos) are the only morphological features of any planar shape. Therefore, the morphology of a polygonal shape is completely characterized by the numbers Δ_J and Δ_T in its DT. Another important morphological descriptor of a shape is its *skeleton* (the 'frame' over which the 'meat' of the shape hangs). According to the morphological meaning of each kind of triangle in the DT of a polygonal shape, an appropriate one dimensional retract can be constructed in each of the triangles, to collectively yield a skeleton of the shape (see Figure 3). This method of constructing skeletons has several advantages over existing methods. More information can be found at http://cnls.lanl.gov/Highlights/1997-07.

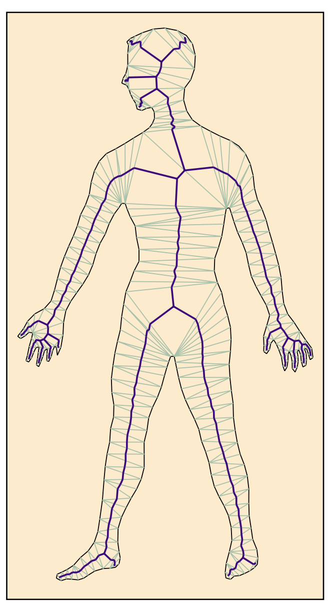


Figure 3.

prasad@lanl.gov
Los Alamos